Multiple Criteria and Multiple Periods Performance Analysis of the Brazilian Public Fundamental School: Employing the DEA-MALMQUIST Method

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This study estimated the efficiency and productivity of the Brazilian public fundamental school, using the production theory with a multiple criteria and multiple periods performance analysis. The multiple criteria approach was based on the method Data Envelopment Analysis (DEA) for measuring the relative efficiency of the unities of the Brazilian federation that operate in differentiated contexts and deal with multiple-inputs/multiple-outputs. The multiple periods analysis was combined using Malmquist Productivity Index (MPI) that measures the total factors productivity change, decomposed into technical efficiency change and technological changes. In this research, education is seen as any other productive function and it is represented by the technical relation between one set of productive factors that are combined in order to produce a given set of outputs. Hence, we selected the variables that influence, determine and represent the school results: the school environment, inputs and outputs of the model. To characterize the environment was selected the Human Development Index of the states (IHD-E). The considered inputs are: a) the average of teaching function per 1000 pupils at the fundamental education from 5th to 8th grade; b) the average of classes per 1000 pupils at 8th grade; c) the number of pupils removed for abandonment by 1000 pupils at 8th grade. The first variable is related to the human resources allocated and the second to the physical infrastructure and equipments available. The last variable in fact is an undesirable output, but as it represents a social outlay, we decided to acknowledge it as an input, inverting its original value. The selected education output were: a) the approval rate at the 8th grade; b) the average proficiency on Portuguese at 8th grade in urban schools; c) the average proficiency on Math of the Saeb at 8th grade in urban schools. The first indicator measures the quantitative aspect of the education process, while the others measure the quality of the output. The results of analysis indicate that the mean of the efficiency indexes of the unities resulted of 97.22%, in which 0.39% is explained by environment factors and 2.39% by the management inefficiency. On the period of the analysis, the MPI reflects an incline that can be explained by the technological innovation. The investigation also demonstrated that there is a strong evidence of the convergence on the development of the unities. An important conclusion is that although the obvious positive correlation between the resources available and the education results, such relation is likely to become spurious as inefficiency evidences comes out. Higher allocation of inputs does not ensure better results, if inefficiency of the educational unities is not solved, i.e. more resources to an inefficient unity will end up on wasting money.

1. Introduction

Human capital is the biggest comparative advantage of the countries, the organizations and the individuals to face the competitiveness, the globalization, the sped up change of the technological bases and the uncertainties of the future. The investment in the formation of the human capital constitutes a crucial aspect for the economic development of the country, the improvement of the exercise of the democracy, the promotion of the equity and the citizenship, as well as for the fall of infantile mortality, the increase of the life expectancy. Unhappily, the resources are scarce and the demands are limitless: beyond a good education, all would like to have a good level of life, good hospitals, public security etc. From there, the necessity to use criteria of economic rationality in the analysis of the allocation of resources in the education, to improve its management and its levels of efficiency.

This study estimated the efficiency, productivity, technological changes and the impact of non-controllable variables in the management of the Brazilian public fundamental school, using a multiple criteria and multiple periods performance analysis.

The multiple criteria approach was based on the non-parametrical method data envelopment analysis (DEA) for measuring the relative efficiency of the public educational systems of the Brazilian federation that operate in differentiated contexts and deal with multiple-inputs/multiple-outputs. DEA is a planning and assessment powerful tool, because it allows define the best practices and needed changes in order for inefficient units to become efficient. It presents several solutions that allow improving efficiency more flexibly and can be used for the identification of input excesses and output shortfalls (slacks), as well as on the formulation of cost reduction policies connected to efficiency optimizing expansion and in the devising of the ideal size for the unities. Furthermore, it permits to identify the likely impact of non-controllable variables and to isolate them in order to have a more precise evaluation of its effects.

The multiple periods analysis was performed using Malmquist productivity index (MPI) combined with the DEA, that measures the total factors productivity change, decomposed into technical efficiency change and technological changes and therefore verifies the convergence or divergence hypothesis of on the developing of the units under analysis. We focus upon the years 2001, 2003 and 2005 employing the data that are used to give an overview of the national education and to serve as reference for formulating policies and implementing programs, including the transfer of public resources.

2. Concepts of Measurements and Estimation

This section presents a summary of the main concepts and non-parametrical tools for estimating efficiency and productivity. We base our thoughts on Álvarez (2001), Cooper, Seiford and Tone (2007) and Färe, Grosskopf e Lovell (1985).

2.1 Technology and the Production Possibilities Set (PPS)

Technology is a key element of the production process. It represents the main restriction on results optimization. It results from the incorporation of scientific knowledge upon productive processes. Nevertheless, it does not only restrain to technical aspects related to production engineer. In a more wide sense, technology is related also to management aspects. In essence, the mean by which inputs are joined and transformed into outputs for producing goods and services is the main feature of a productive process.

The technology is characterized by Production Possibility Set (PPS). In math language, PPS is defined as the set of all m outputs, $y \in R^m$, which can be produced with the vector input $x \in R^n$, that means PPS $=\{(x,y)$: x can produce y; x e $y \geq 0\}$. PPS generates a convex non-negative space R^{n+m} that have two types of axes, namely inputs and outputs. In this sense, each pair of vectors represents a feasible productive process. The frontier of the PPS is defined either by the lowest amount of input able to produce the output vector, or by the highest production level possible for the input vector. This fact indicates that efficient enterprises represent the frontier. If the frontier potential product is defined by f(x), PPS= {y: y ≤ f(x)}, when ≤ suggests the possibility of producing less quantities of y employing the same amount of inputs. Therefore, the productive process is able to reach the same production level employing higher quantities of any x. In other words, the inefficient production below the frontier is very likely to happen. Besides, PPS acknowledge that increases on inputs are likely to generate increased, constant or even decreased returns to scale on the production levels.

There are two main methods to model PPS in order for featuring technology in empirical studies: the parametric and the non parametric. The later is more traditional method and it specifies PPS from the relation between the employed inputs and the maximum amount of outputs produced, by estimating the parameters of the function through econometric techniques. Then, PPS is obtained by the so-called production function (y=f(x)), or other functions associated to it, such as the minimum costs function and the maximum profits function. By considering the process as a relation of equality, the parametric function incurs into some limitations for estimating the technology employed by PPS = {y: y≤ f(x)}, which is defined by an inequality. As pointed out by Ruggiero (1996), if the assumption that production is technically efficient is not valid, parameter estimates may be biased.

A pioneer example of this method for studying public education is Coleman *et al*. (1966). Another classic publication on this subject is Hanushek (1989), who analyzed 147 estimations of the education production function. From study of Hanushek (1986), one can infer that the general presentation for the education production function is the equation $y = F(i, b, p, t, s)$, when y denotes the performance of the pupils, which is associated to several factors grouped into five categories: innate abilities and personal features of the pupils (i), such as gender and race; family background (b), such as income or other socio and economical level measurement; fellow features or peer effect (p); teacher’s features (t), such as experience, salary and scholarship; and other school features (s).

On the non-parametric metric, PPS is defined in a more inductive way, by computing the whole set of technological productive processes. The non-parametric method assumes more flexible hypotheses on the behavior of the variables (distribution) and it does not imply the specification of any functional relationship among inputs and products. The non-parametric method deploys the mathematical programming based upon inequality relations given by PPS={y: y≤ f(x)}. Therefore, it doesn’t requires the traditional hypothesis of efficient behavior of the unities, very important when analyzing the public sector in which the automatic liquidation mechanism doesn’t exist and inefficient organizations are safe from bankruptcy (Pebraja-Chaparro *et al*., 2001). Nonetheless, for estimating a deterministic production frontier, this technique is very susceptible to external observations and it ignores random disturbances of the productive process.

The extant literature does not provide a consensus about which one is the most adequate method. The choice depends on factors such as technology peculiarities, data availability and the research focus. Though, according to Álvarez (2001, p. 32), the non-parametric method is far more employed in investigation than the parametric method is.

2.2 Efficiency Indexes and the DEA models

The development of the non-parametric method was the result of an empirical study in education sector and it assigned to Charnes, Cooper e Rhodes (1978). They labeled the method as Data Envelopment Analysis – DEA.

Behind DEA lays the attempt of measuring the efficiency index proposed by Farrell (1957), which was embedded within Debreu's ideas (1951) and equivalent to Shephard's distance function. The efficiency index of Farrell (F) is calculated by comparing one productive unity with efficient unities that composes the PPS frontier. This index measures the inefficiency from the distance that separates out this unity from the efficient frontier. This measurement implies to a choice for a direction, which can be vertical and horizontal in order to define the existence of two efficiency index types: oriented to inputs and to outputs. The input oriented index (F_i) determines the maximum proportional reduction (radial) of the input vector, given an output vector. For its turn, the output orientation index (F_o) determines the maximum proportional expansion of the output vector keeping constant input vector. Therefore, if a productive process α is at PPS frontier, $F_i(x_\alpha,y_\alpha)= F_o(x_\alpha,y_\alpha)=1$ and α belongs to the efficient subset of PPS. However, if α is inside of PPS, $F_i(x_\alpha,y_\alpha)<1$ and $F_o(x_\alpha,y_\alpha)>1$, then this situation is regarded as inefficient. By multiplying $F_i(x_\alpha,y_\alpha)$ by the vector input or $F_o(x_\alpha,y_\alpha)$ by the vector output, the pair of the vectors (x_α,y_α) is projected to the PPS frontier, allowing α to be regarded as efficient.

Farrell's indexes are reciprocal to Shephard's distance function (1953). This function represents PPS by defining the set of possible outputs [P(x)] of being produced from a given input vector, or the necessary set of inputs [L(y)] for obtaining an output vector. Mathematically, the output oriented distance function can be defined as: $D_o(x,y)=Min\{\theta:(x, y/\theta) \in P(x)\}$, in which the scalar $\theta \in (0,1]$ and it measures the distance from one productive process up to the frontier in the space P(x). The input oriented distance function is defined as $D_i(x,y)=Max\{\delta:(x/\delta,y) \in L(y)\}$, in which $\delta \geq 1$ demonstrates in which proportion inputs are likely to be reduced in the space L(y). When $\theta=\delta=1$, the unit under analysis is regarded as efficient, on the other hand ($\theta<1$ e $\delta>1$) it is likely to be regarded as inefficient. Thus, the relation between the distance function and the Farrell's index is represented as: $D_o(x,y)= [F_o(x,y)]^{-1}$ e $D_i(x,y)= [F_i(x,y)]^{-1}$.

The ideas above lead Farrell to distinguish three types of efficiency: 1) technical efficiency: when the enterprise is within the PPS frontier; 2) allocative efficiency: when the enterprise is able to select the mix of inputs that produces a given quantity of outputs at minimum cost; 3) economic efficiency: when the enterprise is able to achieve the two efficiencies levels at the same time. Among these three efficiencies concepts, the DEA focuses on understanding technical efficiency for depicting the best combination between input and product. It also allows integrating objectives (outputs) that are regarded as non-compatible with the allocative and economic efficiency, for instance: public policies on equity and employment. However, in order to calculate the technical efficiency index, DEA does not employ the Farrell's algebraic method, instead linear programming as demonstrated bellow.

According to Farrell (1957), its approach also determines the difference between "technical efficiency" and "productivity" concepts. Productivity is a measure of output from a production process, per unit of input and shows the levels of how good resource has been employed for each observed productive process. By comparing each productivity level with the best practice, one is able to perceive the efficiency relative concept. Consequently, efficiency can also be labeled as relative productivity. Such index allows defining the needed changes on the levels of input in order to make them efficient. It noted that efficiency is reached when productivity is maximized.

It is worthy to underline that efficiency as a relative value is insensitive to changes on the measurement unities employed to measure inputs and outputs. In order to clarify the matter, we may take into account the assessment of cereal farms. Instead of tone of cereals per hectare, we could employ bushels per acre; the efficiency value would be the same; on the other hand, the productivity would have different values and we will need to specify the measurement unities.

Nonetheless, if we have several inputs and outputs, the productivity calculated employing this rational turns to a partial indicator and it allow to assign to a given input the result generated by other input that we didn't have take into account. The detectable increase of one input can be obtained in spite of diminishing the other and it is due to the existence of compensations on the replacement of inputs and products.

In order to correct this difficult it is necessary to balance inputs and outputs and to replace them by one aggregated value, to the extent that they don't have mutual unities of measurement. Thus, the Total Productivity of the Factors (TPF) emerges. The TPF is defined as the ratio between the weighted total of m outputs and the weighted total of n inputs (x): TPF=$\frac{\sum u_r y_r}{\sum v_i x_i}$, in which u_r Є R^m and v_i Є R^n are the respective weighing that allows to create an aggregated value for y and x.

Given a TPF of one unity, its efficiency assessment requires the determination of the frontier point ($\tilde{y}$) that maximize the outputs with the input vector of the assessed unity. If TPF at this point (x,$\tilde{y}$) is $\frac{\sum u_r \tilde{y}_r}{\sum v_i x_i}$, than the efficiency of the assessed unity will be $=\left(\frac{\sum u_r y_r}{\sum v_i x_i}\right)/\left(\frac{\sum u_r \tilde{y}_r}{\sum v_i x_i}\right) = \frac{\sum u_r y_r}{\sum u_r \tilde{y}_r}$.

However, a question remains: how to properly weight inputs and outputs and what aggregation function needs to be employed on such assessment process? The assessment results are likely to be different according to the existing multiple criteria. The parametric method, for instance, employs market prices to weight the amount of inputs and outputs. This allows calculating TPF and the stochastic production frontier. From this calculation, one can estimate the efficiency index. Another criterion is used by DEA. It employs a so called optimum weighting set {u_r, v_i} derived from linear programming problems, which is known as shadow prices set.

The insensibility of the efficiency radial indexes to the changes on the measurement unities exonerates the employment of market prices. As Cooper *et al.* (2007, p. 39) have proved, if one multiplies inputs (x_{ij}) and outputs (y_{rj}) of N unities analyzed by its respective prices (p_i e p_r >0), the efficiency índex calculated by DEA continues being the same, but weights {u_r, v_i} assume other values: {u_r/p_r, v_i/p_i}.

This knowledge allows the study of multiples outputs and inputs. It also increases the reliability of the model to the extent that prices, affecting and being affected by inputs and outputs, frequently change on time and space and they are distorted weightings. In addition, they incorporate the possible implications of allocative inefficiency to technical efficiency. This is another main reason of DEA popularity, which makes studies on public and non-for-profit organizations feasible. These are typical producers of multiple goods and services within a market free situation, e.g. there is no price as market regulator and monetary weighting is not possible.

Illustration [1] presents the DEA initial model, which has been developed by Charnes *et al.* (1978), oriented to inputs (CCR-OI) for a given unity *o* (from a set of N homogeneous organizations) that produces the vector y_{ro}, and employing the input vector x_{io} and a technology with constant return to scale (CRS). Its calculation involves obtaining the v_i and u_r values (the specific weight or the relative importance of input i and product r) maximizing the efficiency of the unit *o* (H_o), which is defined as the quotient between the weighting sums of outputs and inputs, restricted by the condition that none of the unity have a value higher than one employing the same weights. The implicit restriction, which admits the total flexibility of the weightings credited to inputs and outputs, allows the unities under assessment can combine outputs and inputs differently, which should taken into account for evaluating its inefficiencies. In this vein, the unity under assessment will be compared with the efficient unities set that has its same technological profile.

$$Max\ H_o = \left[\sum_{r=1}^{m} u_r y_{ro}\right] \div \left[\sum_{i=1}^{n} v_i x_{io}\right]$$

$$\text{s. a.} \left[\sum_{r=1}^{m} u_r y_{rj}\right] \div \left[\sum_{i=1}^{n} v_i x_{ij}\right] \leq 1;\ u_r, v_i \geq 0;\ j = 1,.,N;\ r = 1,.,m;\ i = 1,.,n \quad [1]$$

Formulae [1] is transformed into a Linear Programming Problem (LPP) for avoiding infinite solutions and it can be restricted to u_r e $v_i \geq \varepsilon$ (in which $\varepsilon = 10^{-6}$) in order to impede that weights have zero value. That means, it is a solution to avoid that same product or input be left apart from the efficiency determination. The new LPP CCR-OI is given by [2] and its equivalent, LPP CCR-OO, by [3].

$$Max\ H_o = \sum_{r=1}^{m} u_r y_{ro.} \ \ \text{s. a.} \ \sum_{i=1}^{n} v_i x_{io} = 1; \sum_{r=1}^{m} u_r y_{rj} \leq \sum_{i=1}^{n} v_i x_{ij\,;}\, u_r, v_i \geq \varepsilon; \quad [2]$$

$$Min\ \phi_o = \sum_{r=1}^{m} v_i x_{io.}\ \text{s. a.} \sum_{i=1}^{n} u_r y_{ro} = 1\ ;\ \sum_{r=1}^{m} u_r y_{rj} \geq \sum_{i=1}^{n} v_i x_{ij\,;}\ u_r, v_i \geq \varepsilon. \quad [3]$$

These models must result equal. Only the efficient combinations with H=1 on [2] are likely to reach [3] ϕ=1. The inefficient combinations will show ϕ>1, which will have an inverse value to the model [2] (ϕ=1/H). Then, if H_o with [2] is 0.8, with [3] will have ϕ_o=1.25=1/0.8. This will indicate that *o* should, proportionally (radial), to improve outputs on 25% or to reduce inputs on 20% in order to be regarded as efficient.

The indexes are calculated by [2] can be represented as in Figure 1. It illustrates a frontier for the input space of PPS, devised by the linear combination of the unities that minimize the inputs x1 and x2 needed to produce one unity of y. As demonstrated above, any point within the frontier is efficient and has H=1. Then, no reduction on unities C and B is viable to maintain the production level. Unities above the frontier are regarded as inefficient. Therefore, H_F<1, being F' (the linear combination of points C and B or the radial projection of F up to the frontier) the value that inputs must assume in order to F be regarded as efficient.

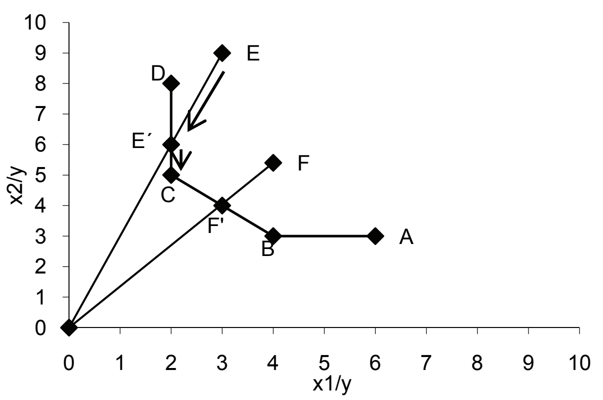

Figure 1- Efficient Frontier, Inputs Orientation

On Figure 1, one can observe that the determination of the frontier by LPP results into linear segments. The parallel to the axes allows identifying productive process slacks, when one of the weights (v_i or u_r) are equals to zero. For instance, the unities (D, E' e A) located at the vertical segment of the frontier has H=1, though their performance are likely to be improved by reducing parts of one input, without reducing the production level.

For illuminating this situation, Cooper *et al*. (2007, p. 45) have formulated the following concept: one unity is efficient if and only if its index is equal to one and all slacks are equal to zero; on the contrary, such unity must be regarded as inefficient. This definition corroborates Koopmans, which, following Pareto's principle, define that: one manager that employs two or more inputs for producing one product is efficient only if he/she is able to reduce the consumption of one input while levering the use of others. These formulations are far more restrictive than Farrell's definition. Therefore, on Figure 1, the section BC is labeled as Pareto-Koopmans's frontier, or strongly efficient frontier. The section of the frontier from B to C, including the section BA and the vertical line above C is defined as Debreu-Farrell frontier, or also weakly efficient frontier. In this sense, the point E must be projected to E', regarded efficient according to Farrell's concepts, because E' is located exactly upon the frontier. This is not true on Pareto-Koopmans' concept, because the input x2 could still be reduced up to the level of C, without changing the production level. The same would happen if D moves to C and A moves to B; this reduction is called as slack improvement (s_i^-) and it represented by a non radial dislocation.

The same logic applies for explaining the indexes calculated by [2] and for identifying efficiency and slacks in the output space of PPS.

The projection of the inefficient unities upon the strongly efficient frontier, the redundancy of inputs (s_i^-) and the slacks of products (s_r^+) can be calculated by [2] and [3] in a residual way. Though, it can be calculated straightly through the dual problems of these formulations, which are called two stage models. The Dual formulation of CCR-OI is given by [4] and the Dual of CCR-OO by [5]. Note that in both models H_o, ϕ_o, s_i^-, s_r^+ e λ_j are calculated. The last symbol (λ_j) represents the linear combination coefficients of the best practices that determine the projection of the inefficient unities upon the strongly efficient frontier.

$$Min\left[H_o - \varepsilon\left(\sum_{i=1}^{n} s_i^- + \sum_{r=1}^{m} s_r^+\right)\right]$$

$$s\ a: \sum_{j=1}^{N} \lambda_j y_{rj} - s_r^+ = y_{ro}; \sum_{j=1}^{N} \lambda_j x_{ij} + s_i^- = H_o x_{io};\ \lambda_j,\ s_i^-, s_r^+ \geq 0. \qquad [4]$$

$$Max\left[\phi_o - \varepsilon\left(\sum_{i=1}^{n} s_i^- + \sum_{r=1}^{m} s_r^+\right)\right]$$

$$s\ a: \sum_{j=1}^{N} \lambda_j y_{rj} - s_r^+ = \phi_o y_{ro}; \sum_{j=1}^{N} \lambda_j x_{ij} + s_i^- = x_{io};\ \lambda_j,\ s_i^-, s_r^+ \geq 0. \qquad [5]$$

In this way, the unities of reference compose a so-called efficient subset $\{j: \lambda_j^* > 0\}$ and they define the improvement targets ($\hat{x}_{ij}$, $\hat{y}_{rj}$) of the inefficient by employing the following formulae:

Orientation to inputs [4.1]
$\hat{x}ij = H_j^* x_{ij} - s_{ij}^{-*} = \sum x_{ij}\lambda_j^*.$ $\hat{y}rj = y_{rj} + s_{rj}^{+*} = \sum y_{rj}\lambda_j^*.$
Orientation to outputs [5.1]
$\hat{y}rj = \phi_j^* y_{rj} + s_{rj}^{+*} = \sum y_{rj}\lambda_j^*.$ $\hat{x}ij = x_{ij} - s_{ij}^{-*} = \sum x_{ij}\lambda_j^*.$

As demonstrated before, the CCR model presupposes that the unities work according to constant returns to scale (CRS). In face of the competition, most of the productive sector operates at CRS, which determines the optimum size and the higher productivity. However, in imperfect competition situations, as in the public sector, there are a possibility of operating on variable returns (increasing and decreasing) to scale (VRS), therefore, the DEA's initial model has been expanded by Banker, Charnes e Cooper (1984) for including VRS technologies. The new model, labeled as BCC, evaluates the set of unities depending on the nature of the returning to scale and it allows comparing different sizes of unities.

The BCC-IO dual [6] is calculated by adding to the dual model CCR-IO dual [4] one restriction for ensuring that the unity under analysis will be compared to a convex combination of efficient unities set taking into account the three returns to scale types. The difference between these two measurements is labeled as inefficiency of scale (SE), which can be interpreted as the portion of the inefficiency present on the model [4] stems from the inadequate size of the unity under analysis.

$$Max\left[\tau_o - \varepsilon\left(\sum_{i=1}^{n} s_i^- + \sum_{r=1}^{m} s_r^+\right)\right]$$

$$s\ a{:} \sum_{j=1}^{N} \lambda_j y_{rj} - s_r^+ = y_{ro};\ \sum_{j=1}^{N} \lambda_j x_{ij} + s_i^- = \tau_o x_{io};\ \sum_{j=1}^{N} \lambda_j = 1;\ \lambda_j,\ s_i^-, s_r^+ \geq 0. \qquad [6]$$

Thus, the inefficiency indexes can be seen under different concepts: a) efficiency with CRS, labeled as productive efficiency (PE); b) efficiency with VRS, regarded as pure technical efficiency (PTE); and c) the relation between the formers efficiencies defined as efficiency of scale: SE=PE/PTE. The PE captures the technical inefficiency when the maximum productivity is not achieved, but also incorporates the likely effect of one inadequate production scale of a technology with RVE. In this way, PE can derive from the origin and it can be either pure or of scale. PTE allows isolating the productive inefficiency caused by the technical inefficiency *stricto sensu*, by eliminating the component of a inadequate production scale. SE helps on identifying the distance between frontiers PE e PTE.

In the case of seeking to maximize the production level given levels of input, the BCC-OO dual is given by [7].

$$Max\left[\theta_o - \varepsilon\left(\sum_{i=1}^{n} s_i^- + \sum_{r=1}^{m} s_r^+\right)\right]$$

$$s.a. \sum_{j=1}^{N} \lambda_j y_{rj} - s_r^+ = \theta_o y_{ro}; \sum_{j=1}^{N} \lambda_j x_{ij} + s_i^- = x_{io};\ \sum_{j=1}^{N} \lambda_j = 1;\ \lambda_j,\ s_i^-, s_r^+ \geq 0. \qquad [7]$$

Another adaptation of the DEA model has occurred by the need for introducing on the analysis uncontrollable inputs and environment factors, e.g. variables that are expected to intervene and to influence the productive, but not possible of alterations and out of management control. Though these variables are of different nature, they are not easily distinguished and the separation line between them is of researcher discretion. According to Muñiz (2001), seldom non discretion inputs and environment factors are pinpointed on DEA articles and, therefore, in this paper they will be characterized as uncontrollable variables.

Examples of uncontrollable inputs are cognitive abilities, attitudes, other inmate characteristics of pupils, which are strongly associated with school environment factors, e.g. parents' engagement and the socioeconomic environment in which the pupil lives within. Both are very interrelated and though uncontrollable they show up, in same investigations, more important than the controllable factors on education performance (Coleman *et al.*, 1966).

Models that include these variables in their analysis are able to identify and to isolate impact as well as to assess the efficiency from managing controllable variables and therefore enabling a homogeneous and a fair evaluation of the unities. This seems to be the only way for identifying whether a low performance manager is in fact inefficient or there are other factors that influence performance. According to Muñiz (2001), this can be achieved by employing the three stages Fried and Lovell's model (1996 as quoted by Muñiz, 2001).

At the first stage, it proposes the calculation of the model [6], contemplating discretionary inputs and outputs disregarding uncontrollable variables. The model produces the efficiency index for each unity and the total needed improvements of each input ($S_i^-=(1-\tau_j)x_i+s_i^-$) and of each output ($S_r^+=s_r^+$). The identified improvements (radial and non radial – total slack) derived from the two distinct factors: the inefficiency of the management of the controllable variables and the influence of the controllable variables. In order to isolate these factors, the Fried and Lovell's model indicates two methodological alternatives in one second stage: the econometric method (in which inputs and outputs total slack of the first stage is estimated according to the uncontrollable variables) or the DEA, which is the option this work adopts.

At the second stage it is recommended to run by separate one second BCC-OI [6] for the slack of each variable of the first stage, i.e. it is suggested to calculate as many LPP how many controlled inputs and outputs exist. The model is formulated in order to determine (for each concrete variable of each unity) the maximum possible reduction of the total slack, given the vector for the uncontrollable variables.

According to Muñiz (2001), the logic behind second stage can be illustrated by Figure 2. The goal is to estimate the frontier formed by total possible minimum slack for each variable representing the unities analyzed. This minimum is likely to be explained exclusively by the uncontrollable variable effect. This is the case of the unity A of Figure 2, which has the minimum total slack of S_{ixa}. The unity B achieves a higher slack (S_{ixb}). For calculating the over plus, B is compared with A that has the same value on the uncontrollable variable. Thus, the total slack of B can be discriminated into two: the slack due to the inefficiency on the management of the controllable variables and the slack derived from the influence of the uncontrollable variables. In other words, in the case of a minimum slack coinciding with the value detected at the first stage, it is likely to be explained by the uncontrollable variable effect and, therefore, if this minimum is lower than the initial total slack, one can assume that the excess can be explained by the existence of some inefficiency of the management process.

The analysis of the slack allows adjusting the original inputs and outputs from inefficient unities at first stage. The adjustment consists on improving (or reducing) the outputs (inputs) value of the unities according to the correspondent amount of the minimum slack attributed to the uncontrollable effects, and it allows compensating this exogenous effect. For instance, in the case of unity B of Figure 2, the adjustment on the controllable input will be $x^*_{ib}=x_{ib}-S_{xia}$.

At third stage, the DEA model BCC-OI [6] is recalculated with the amended values. The final efficiency indexes correspond to the manager performance.

2.3 The Malmquist productive index (MPI)

The efficiency index analysis so far developed is static, since variables are employed and unities are compared in a given period. The introduction of a temporal dimension allows creating a dynamic model, which moves the central question to other very important problems: 1) the evolution of each unity productivity in relation to the evolution of the set of the assessed unities; 2) the analysis of the productivity index in terms of the technological changes, pure technical efficiency; and 3) the feature of the temporal path (with or without flotation, with a convergence or divergence tendency).

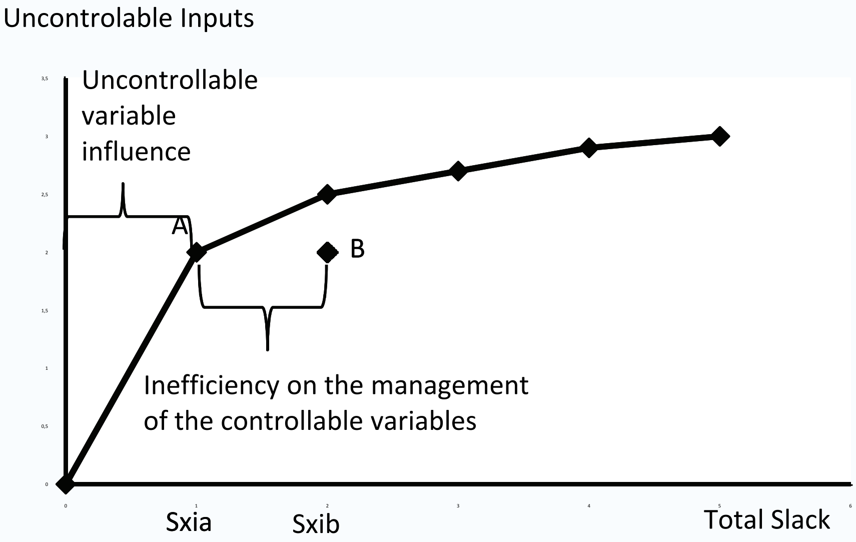

Figure 2- Total Slack Analysis on second stage
Fonte: Adapted from Muñiz (2001)

In order to assess the productivity evolution, four indexes are frequently used: the geometric index of Divisia, the Fisher Index, the Törnqvist index, and the Malmquist productivity index (MPI). The three former are associated to market prices and to the parametric method. The MPI is associated to the non-parametric method, although it also works with parametric techniques (Marinho *et al*., 2000).

The MPI refers to the work of Sten Malmquist published in 1953 and it is a quantity index based on distance functions within the consumer theory. Later, Caves, Christensen e Diewert (1982) inserted this index into the production theory context and defined the MPI oriented to inputs and to outputs, but they did not relate it to the Farrell index. Färe, Grosskopf, Lindgren and Roos's (1992) were the first to combine the DEA with the MPI.

Given any unity of a group of organizations that produce the vector y^t employing the input vector x^t and the technology CPP^t in one particular period of time t, Caves *et al*. (1982) defined the MPI as the ratio between the distance functions of the unity in two distinct periods (t e t+1) taking as reference the technology (frontier) of the t period. Considering the technology - superscript and the orientation - subscript, the MPI oriented to inputs and to outputs are respectively:

This formulation conveys the evolution of the efficiency of the unity assessed on period t to t+1 in relation to the best practice of the period t. Given that one efficient frontier is taken as reference, the MPI demonstrates the performance of the total factors productivity (TFP). As 1: a) figures higher than one indicates that the TFP of the period t+1 has improved in relation to the period t (a reduction of the distance between the observed production and the potential product in t is observed); b) figures equal to one indicates that the distance is constant between t e t+1; and c) figures lower than one indicates the TFP is worse than before.

The MPI is possible to be defined also assuming that the technology of the period t+1, that is likely to be dislocated upwards as time passes by due to technical and organizational innovations, i.e. technological innovations. This index will not necessarily be the same as its antecessor. Moreover, on attempting to overcome such divergence and following the tenets of the ideal index of Fisher, the MPI can be defined as the geometric mean between MPI^t e MPI^{t+1}, in other terms,

$$MPI_i^{t,t+1}(x^{t+1},y^{t+1},x^t,y^t) = [MPI_i^t(x^{t+1},y^{t+1},x^t,y^t) * MPI_i^{t+1}(x^{t+1},y^{t+1},x^t,y^t)]^{1/2} = $$
$$= [[D_i^t(x^{t+1},y^{t+1})/D_i^t(x^t,y^t)] * [D_i^{t+1}(x^{t+1},y^{t+1})/D_i^{t+1}(x^t,y^t)]]^{1/2} \quad [10]$$
$$MPI_o^{t,t+1}(x^{t+1},y^{t+1},x^t,y^t) = [MPI_o^t(x^{t+1},y^{t+1},x^t,y^t) * MPI_o^{t+1}(x^{t+1},y^t,x^t,y^t)]^{1/2} = $$
$$= [[D_o^t(x^{t+1},y^{t+1})/D_o^t(x^t,y^t)] * [D_o^{t+1}(x^{t+1},y^{t+1})/D_o^{t+1}(x^t,y^t)]]^{1/2} \quad [11]$$

The MPI calculation involves four distance functions. Considering that the distance function is equal to the reciprocal of the Farrell's efficiency index calculated by DEA, the distance function oriented to outputs can be represented by: $[D_o^t(x^t,y^t]^{-1} = PE_t^t$, $[D_o^t(x^{t+1},y^{t+1}]^{-1} = PE_{t+1}^t$, $[D_o^{t+1}(x^t,y^t]^{-1} = PE_t^{t+1}$ e $[D_o^{t+1}(x^{t+1},y^{t+1}]^{-1} = PE_{t+1}^{t+1}$, in which PE_t^t is the productive efficiency for the period given complying with the efficiency frontier of the period given.

Thus, from the equation [11], one can deduce [12] (the definition of the orientation to inputs is similar).

$$MPI_o^{t,t+1}(x^{t+1},y^{t+1},x^t,y^t) = [[PE_t^t/PE_{t+1}^t] * [PE_t^{t+1}/PE_{t+1}^{t+1}]]^{1/2} \quad [12]$$

Färe *et al.* (1992), making the following manipulation have arrived to the following formulation:

$$MPI_o^{t,t+1}(.) = \left[\frac{D_o^t(x^{t+1},y^{t+1})}{D_o^t(x^t,y^t)}\frac{D_o^t(x^t,y^t)}{D_o^t(x^t,y^t)} * \frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^{t+1}(x^t,y^t)}\frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\right]^{1/2}$$
$$= \frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^t(x^t,y^t)}\left[\frac{D_o^t(x^{t+1},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\frac{D_o^t(x^t,y^t)}{D_o^{t+1}(x^t,y^t)}\right]^{1/2} \quad [13]$$

This formulation can be expressed in terms of the productive efficiency and defined as: $MPI_o^{t,\ t+1}(x^{t+1},y^{t+1},x^t,y^t) = \frac{PE_t^t}{PE_{t+1.}^{t+1}}\left[\frac{PE_{t+1}^{t+1}}{PE_{t+1}^t} * \frac{PE_t^{t+1}}{PE_t^t}\right]^{1/2}$ [14]

In this way, the MPI, differently from the other indexes indicated before, allows the productive efficiency evolution to be separated from the dislocations of the frontier. The first ratios on the right side of equation [13] and [14] $\left(\frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^t(x^t,y^t)} = \frac{PE_t^t}{PE_{t+1.}^{t+1}}\right)$ measures how distant the assessed unity is from the efficient frontier between the periods t e t+1. They capture the productive efficiency evolution, an effected known as *catching-up*. They result from the individual capacity of the assessed unity for managing the incorporation of the technical and organizational progress to the productive process. They are likely to be smaller, equal or even higher according to the reduction, stability or improvement on the productive efficiency respectively. The second ratios of the equations [13] and [14] seize the average geometric dislocation of the technological frontier between the two periods under assessment in relation to the level of inputs x^t e x^{t+1}. They represent the technological change (technical and organizational). If the dislocation is higher than one, it is likely to indicate a progress resulting from the innovation in the sector; if it is lower, it is likely to represent a throwback.

Consequently, TFP performance can de decomposed into two independent components, namely productive efficiency change (ECH) and technological change (TCH). Both are likely to evolve in opposite directions, invalidating one to another; or going into the same direction joining forces.

Knowing the relative value of these two determinant factors supplies an important issue to the government decision-making process. Thus, for instance, policies directed to the improvement of the efficiency, eliminating institutional barriers to the transfer and to the diffusion of modern technologies is likely to generate better productivity results and increases that other policies oriented to promote technological innovations (Orea, 2001).

While the cost of copping technologies was lower than the cost of innovation, the unities (countries, organizations, administrative unities, programs, so on and so forth) are likely to get closer of the leaders supporting the convergence hypothesis. According to this hypothesis, underdeveloped unities are likely to grow faster on time rather than the efficient and developed unities, because the later are only able to acquire TFP improvements making technological innovations. In this way, the divergent tendencies hypothesis of the economic development is questioned (Marinho, *et al*., 2001).

The analysis of the MPI still can be expanded in order to include VRS employing the BCC model. According to what was established above, the result of such expansion is the decomposition of the productive efficiency change (ECH) as the pure technical efficiency changes (PTECH) and the scale of efficiency changes (SECH). That means, according to Färe, Grosskopf, Norris and Zhang (1994):

$$ECH = \frac{D_O^{t+1}(x^{t+1}, y^{t+1})|\text{CRS}}{D_O^t(x^t, y^t)|CRS} = \frac{PE_t^t}{PE_{t+1}^{t+1}}$$

$$= \frac{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}{D_O^t(x^t, y^t)|VRS} \frac{\frac{D_O^{t+1}(x^{t+1}, y^{t+1})|CRS}{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}}{\frac{D_O^t(x^t, y^t)|CRS}{D_O^t(x^t, y^t)|VRS}} = \frac{PTE_t^t * SE_t^t}{PTE_{t+1}^{t+1} * SE_{t+1}^{t+1}} \quad [15]$$

In which: PTECH$=\frac{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}{D_O^t(x^t, y^t)|VRS} = \frac{PTE_t^t}{PTE_{t+1}^{t+1}}$ represents the relation of the distance functions assessed into a technology under VRS. PTECH$\lesseqgtr 1$ and according to the reduction, stabilization or improvement of the pure technical efficiency between the periods t and t+1.

SECH$=\frac{\frac{D_O^{t+1}(x^{t+1}, y^{t+1})|CRS}{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}}{\frac{D_O^t(x^t, y^t)|CRS}{D_O^t(x^t, y^t)|VRS}} = \frac{SE_t^t}{SE_{t+1}^{t+1}} = \frac{\frac{PE_t^t}{PTE_t^t}}{\frac{PE_{t+1}^{t+1}}{PTE_{t+1}^{t+1}}}$ expresses the changes of the operation scales in relation to the optimum size. Therefore, SECH>1 means that the unity under assessment is closer to the optimum production scale on the period t+1, and far from if MEE<1.

Consequentently, the complet version of MPI is given by:

$$MPI_O^{t,t+1}(x^{t+1}, y^{t+1}, x^t, y^t)$$

$$= \frac{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}{D_O^t(x^t, y^t)|VRS} \frac{\frac{D_O^{t+1}(x^{t+1}, y^{t+1})|CRS}{D_O^{t+1}(x^{t+1}, y^{t+1})|VRS}}{\frac{D_O^t(x^t, y^t)|CRS}{D_O^t(x^t, y^t)|VRS}} \left[\frac{D_O^t(x^{t+1}, y^{t+1})|CRS}{D_O^{t+1}(x^{t+1}, y^{t+1})|CRS} \frac{D_O^t(x^t, y^t)|CRS}{D_O^{t+1}(x^t, y^t)|CRS}\right]^{(}$$

$$= \frac{PTE_t^t}{PTE_{t+1}^{t+1}} \frac{SE_t^t}{SE_{t+1}^{t+1}} \left[\frac{PE_{t+1}^{t+1}}{PE_{t+1}^t} * \frac{PE_t^{t+1}}{PE_t^t}\right]^{1/2} = ECH * TCH = PTECH * SECH * TCH \quad [16]$$

With this decomposition one can determine that the components (PTECH, SECH e TCH) compose the main source of increase or decrease of the TFP.

3. The Performance of the Brazilian Education at the 8th Grade

The goal of this section is to demonstrate the application of the method illustrated in section above. For doing so, we have chosen the fundamental education on its final stage (the 8th grade) for the years 2001, 2003 and 2005 for the 27 unities of the Brazilian federation. That means, we focused on the country as a whole and the five geographic regions, namely south, southeast, central, north, and northeast.

The investigation of the educational unities performance demands the definition of a conceptual model in order to understand its operation. In this investigation, education is seen as any other productive function and it is represented by the technical relation between one set of productive factors that are combined in order to produce a given set of outputs.

Data has been collected from Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira - INEP (2005): the Educational Census and the Basic Education System Evaluation (Saeb). From this data, we selected the variables that influence, determine and represent the public school results : the educational environment, inputs, and outputs of the model .

As in other countries, the variation o the performance of Brazilian schools relates to differences on the environment of these schools. In general, the school environment is featured by a large set of uncontrollable variables of related to the individual, economic, social and cultural nature, such as the socioeconomic level of the family, the scholarity level of the parents, the motivation level of the family members and of the class, and the relationship between home and school. These set needs to be simplified into coherent criteria and factor analysis are often employed for doing so. As we do not have this information available, in this investigation we have employed a proxy: the Human Development Index of the states (HDI_E), which is detailed on the CEPAL Report (2008). It reasonably summarizes the environment of the schools and it is also correlated to several uncontrollable inputs.

The considered inputs are: a) the average of teaching function per 1000 pupils at 5th - 8th grade (I_1); b) the average of classes per 1000 pupils at 8th grade (I_2); c) the number of pupils removed for abandonment by 1000 pupils at 8th grade (I_3). The first variable is related to the human resources allocated and the second to the physical infrastructure and equipments available. The last variable in fact is an undesirable output, but as it represents a social outlay, we decided to acknowledge it as an input, inverting its original value. This is a method suggested by Gomes and Lins (2008) for treating undesirable outputs.

In terms of choosing outputs we shall remark that it is a complex decision due to difficulties on the delimitation of the multidimensional and global nature of educational objectives. In this sense, the literature has shown a prominence of important but intermediate results of the education process, namely school grades. In this way, the selected education output were: a) the approval rate at the 8th grade (O_1); b) the average proficiency on Portuguese at 8th grade in urban schools (O_2); c) the average proficiency on Math of the Saeb at 8th grade in urban schools (O_3). The first indicator measures the quantitative aspect of the education process, while the others measure the quality of the output.

3.1 Main results of the analysis

The chosen variables allow us to calculate the efficiency indexes BCC-OI dual [6] of the assessed unities. These indexes are presented on columns 2, 3 e 4 in Table 1 and they evidence of the best management practices in absolute terms disregarding the uncontrollable variables (context conditions). Twelve states are at the top of the ranking.

However, as explained before, it is necessary to address the impact of uncontrollable variables featured on HDI_E. We suppose that the HDI_E affects the scholar production in the extent that their influence is very likely to influence the efficiency índex of the states. School managers are not to be held responsible for inefficiencies that they are not able to control. Therefore, we shall not use these indexes and the three stages Fried-Lovell model is to be employed as explained elsewhere.

Then, after finishing the first stage, we immediately started the second employing the total slack found as an input and the HDI_E as an output as in BCC-OI dual model [6]. It allows determining the effect of the uncontrollable variable to correct the original inputs and outputs, at the 3rd stage, estimating the value that grasp exclusively the efficiency of school managers. These indexes are on the last columns of Table 1

The comparison among the indexes of Table 1 allows reaching some interesting conclusions. First, the increasing of the average corrected indexes is observed. One can perceive, at the last line of Table 1, a decrease on the standard deviation, which confirms that corrected indexes are more homogeneous. The aggregated geometric averages of the three periods increased from 97.22% of the original data to 97.61% of the corrected ones. This fact indicates that 0.39% of the original inefficiency is likely to be explained by the impact of the exogenous variable and 2.39% by the managerial inefficiency. The role of surrounding (14.06%) on the inefficiency levels is evidenced. Secondly, the number of efficient states has increased. Now, the list reaches 14 states and two regions regarded as efficient. The other states (with the exception of Tocantins, Roraima and Piauí) have averages higher than 90%.

The BCC-OO model [7], now with the corrected figures presents similar results. The efficient states are the same of the former model and Tocantins is still the most inefficient with a geometric average of 103.7. The aggregated geometric average result for the three periods was 100.73%, and this indicates that if the same good practices were adopted by all states, the scholar results would be superior in 0.73%.

Table 1: BCC-OI Efficiency Indexes with Original and Amended Data

Unity	1st stage			3rd stage		
	Index 2001	Index 2003	Index 2005	Index 2001	Index 2003	Index 2005
Brazil	0.96	1.00	0.96	0.97	1.00	0.97
North	0.95	0.95	0.98	0.96	0.96	0.99
Rondônia	0.95	0.84	1.00	0.96	0.84	1.00
Acre	0.81	1.00	1.00	0.82	1.00	1.00
Amazonas	1.00	1.00	1.00	1.00	1.00	1.00
Roraima	0.83	0.95	0.84	0.84	0.95	0.86
Pará	1.00	1.00	1.00	1.00	1.00	1.00
Amapá	0.88	1.00	1.00	0.89	1.00	1.00
Tocantins	0.92	0.83	0.91	0.93	0.83	0.92
Northeast	0.96	1.00	0.95	0.99	1.00	0.96
Maranhão	0.88	1.00	0.99	0.90	1.00	1.00
Piauí	0.89	0.85	0.91	0.91	0.85	0.92
Ceará	1.00	1.00	1.00	1.00	1.00	1.00
R. G. do Norte	0.93	0.96	1.00	0.93	0.96	1.00
Paraíba	0.93	0.96	1.00	0.93	0.96	1.00
Pernambuco	0.99	1.00	1.00	0.99	1.00	1.00
Alagoas	1.00	1.00	1.00	1.00	1.00	1.00
Sergipe	0.95	0.92	1.00	0.96	0.92	1.00
Bahia	1.00	1.00	0.94	1.00	1.00	0.95
Southeast	1.00	1.00	1.00	1.00	1.00	1.00
Minas Gerais	1.00	1.00	1.00	1.00	1.00	1.00
Espírito Santo	0.98	1.00	1.00	1.00	1.00	1.00
Rio de Janeiro	1.00	1.00	1.00	1.00	1.00	1.00
São Paulo	1.00	1.00	1.00	1.00	1.00	1.00

South	0.92	1.00	0.99	0.99	1.00	1.00
Paraná	0.95	1.00	0.97	0.96	1.00	0.98
Santa Catarina	1.00	1.00	1.00	1.00	1.00	1.00
R. G. do Sul	1.00	1.00	1.00	1.00	1.00	1.00
Central	0.99	1.00	1.00	1.00	1.00	1.00
M. G. do Sul	1.00	1.00	1.00	1.00	1.00	1.00
Mato Grosso	1.00	1.00	1.00	1.00	1.00	1.00
Goiás	1.00	1.00	1.00	1.00	1.00	1.00
Distrito Federal	1.00	1.00	1.00	1.00	1.00	1.00
Average	0.9598	0.9773	0.9830	0.9672	0.9778	0.9866
Standard deviation	0.0510	0.0482	0.0354	0.0480	0.0479	0.0313

Source: Data Analysis

Other important result is the dynamic comparison in which each unity has it TFP assessed in relation to the overall TFP evolution. From the formulation [16] and with the amended figures, we calculated the productive efficiency change (ECH), the pure technical efficiency changes (PTECH), the scale of efficiency changes (SECH), the technological change (TCH) and the Malmquist productivity index (MPI), which are presented on Table 2. In such analysis, the figures lower than 1 indicate a decrease, figures higher than 1 indicate an increase and figures equal to 1 demonstrate that no changes were observed on TFP.

Table 2: ECH, PTECH, SECH,TCH and MPI Oriented to Outputs (Periods 2001-03 and 2003-05)

Unity	Period 2001-03					Period 2003-05				
	ECH	PTECH	SECH	TCH	MPI	ECH	PTECH	SECH	TCH	MPI
Brazil	0.98	0.98	0.99	1.05	1.02	1.01	1.00	1.01	1.01	1.02
North	0.98	1.00	0.98	1.05	1.04	1.02	1.01	1.01	1.01	1.03
Rondônia	0.92	1.03	0.90	1.10	1.02	1.14	1.04	1.09	0.93	1.06
Acre	1.17	0.94	1.25	0.93	1.09	0.99	1.00	0.99	1.02	1.01
Amazonas	1.00	1.00	1.00	0.94	0.94	0.99	1.00	0.99	1.08	1.07
Roraima	0.93	0.96	0.96	1.17	1.08	1.09	0.98	1.12	0.96	1.05
Pará	0.96	1.00	0.96	1.08	1.03	1.02	1.00	1.02	1.01	1.03
Amapá	1.01	0.96	1.05	1.02	1.03	1.11	1.00	1.11	0.91	1.01
Tocantins	0.91	1.02	0.89	1.14	1.03	1.10	1.02	1.08	0.93	1.03
Northeast	1.00	1.00	1.01	1.06	1.07	0.92	0.99	0.93	1.07	0.99
Maranhão	1.13	0.92	1.23	1.01	1.14	0.93	1.00	0.93	1.10	1.02
Piauí	0.91	1.01	0.90	1.19	1.09	1.00	1.02	0.98	1.01	1.01
Ceará	1.00	1.00	1.00	1.07	1.07	1.00	1.00	1.00	1.04	1.04
R. G. do Norte	0.99	0.99	1.00	1.06	1.04	1.03	1.01	1.03	1.00	1.03
Paraíba	1.00	0.99	1.01	1.03	1.03	1.02	1.02	1.00	0.98	1.00
Pernambuco	1.01	0.99	1.01	0.99	1.00	1.00	1.00	1.00	1.03	1.03
Alagoas	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.02	1.02
Sergipe	0.95	1.01	0.95	1.08	1.03	1.08	1.02	1.06	0.96	1.04
Bahia	0.93	1.00	0.93	1.24	1.16	0.99	0.98	1.01	1.03	1.02
Southeast	0.99	1.00	0.99	1.03	1.02	1.00	1.00	1.00	1.01	1.01
Minas Gerais	0.99	1.00	0.99	1.02	1.01	1.01	1.00	1.01	1.02	1.03
Espírito Santo	0.97	1.00	0.97	1.07	1.04	0.99	1.00	0.99	1.02	1.01
Rio de Janeiro	0.97	1.00	0.97	1.06	1.03	1.02	1.00	1.02	0.99	1.00
São Paulo	1.00	1.00	1.00	1.03	1.03	1.00	1.00	1.00	0.99	0.99
South	0.96	1.00	0.97	1.06	1.02	1.01	1.00	1.01	0.99	1.00
Paraná	0.99	0.98	1.01	1.03	1.02	1.00	0.99	1.01	0.99	0.99
Santa Catarina	1.04	1.00	1.04	1.06	1.11	0.99	1.00	0.99	1.01	1.00
R. G. do Sul	0.96	1.00	0.96	1.07	1.03	0.96	1.00	0.96	1.05	1.01
Central	1.01	1.00	1.01	1.01	1.02	1.00	1.00	1.00	1.03	1.03
M. G. do Sul	0.96	1.00	0.96	1.07	1.02	1.00	1.00	1.00	1.02	1.02
Mato Grosso	1.00	1.00	1.00	1.00	1.00	1.11	1.00	1.11	0.92	1.02

Goiás	1.05	1.00	1.05	0.97	1.02	0.97	1.00	0.97	1.05	1.02
Distrito Federal	1.00	1.00	1.00	1.02	1.02	1.00	1.00	1.00	0.96	0.96
Geometric mean	0.99	0.99	1.00	1.05	1.04	1.01	1.00	1.01	1.00	1.02

Source: Data Analysis

According to Table 2, most of the states presented a increase on their MPI in the period 2001-2003, and Amazonas achieved the worse result with a variation of -5.98%. The aggregated geometric average indicates a rise of 4% on the productivity. This increase can be explained by the positive evolution of technology (5%), whose effect was minimized by the negative evolution of productive efficiency. On the period 2003-2005, the analysis of the aggregate geometric average on Table 2 also indicates an increase on productivity variation, and this analysis demonstrates an increase of 2%, which can be explained by the positive variation of the productive efficiency index (1%) and by technological innovations (1%). We can also observe that the majority of states presented increases on their productivity indexes. Only three states presented negative results and they are: São Paulo, Paraná e Distrito Federal.

In order to test the convergency and divergency hypotheses for the 33 unities under analysis, we calculated the Pearson and Spearman's rank correlation coefficient employing the two geometric averages: the HDI_E (as an independent variable) and the MPI – calculated with the original data and therefore without discounting the impact of HDI_E. The both tests revealed significant negative associations between the two variables: Pearson Correlation =-.504 with Sig .003 and Spearman's rho =-.491 with Sig. .004. That means, unities with low HDI_E are likely to increase, on average and on the three years period under analysis, on faster rates than the developed unities are able to do. This is a strong evidence of the convergence tendency in the development of the 33 unities analyzed. Probably, this fact can be explained by the diffusion of modern educational Technologies, which are expressed at the positive changes on the productive efficiency (ECH) observed on Table 2.

Conclusions

An important conclusion this investigation allows to reach is that although the obvious positive correlation between the resources available and the education results, such relation is likely to become spurious as inefficiency evidences comes out. Higher allocation of inputs does not ensure better results, if inefficiency of the educational unities is not solved. In other words, more resources to an inefficient unity will end up on wasting money.

List of References

Álvarez, A. (Ed.) (2001). La Medición de la eficiencia y la productividad. Madrid: Ed. Pirámide.

Banker, R.D., Charnes, A. & Cooper, W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. Management Science, *30*(9), 1078-1092.

Caves, D.W., Christensen, L.R. & Diewert, W.E. (1982). The economic theory of index numbers and the measurement of input, output, and productivity. Econometrica, *50*(6), 1393-1414.

Comissão Econômica para a América Latina e o Caribe - CEPAL. (2008). Emprego, desenvolvimento humano e trabalho decente: a experiência brasileira recente. Brasília: CEPAL/PNUD/OIT.

Charnes, A., Cooper, W. & Rhodes, E. (1978). Measuring the efficiency on decision marking units. European Journal of Operational Research, *2*(6), 429-444.

Coleman, J. S. et al. (1966). Equality of educational opportunity. Washington, D.C.: U.S. Government Printing Office.

Cooper, W., Seiford, L.M. & Tone, K. (2007). Data Envelopment Analysis: a comprehensive

text with models, applications, references and DEA-solver software. Boston: Kluwer Academic Publishers.
Debreu, G. The coefficient of resource utilization. Econometrica, *19*(3), 273-292, 1951.
Färe, R., Grosskopf, S. & Lovell, C. A. K. (1985). The Measurement of Efficiency of Production. Boston-Dordrecht-Lancaster: Kluswer-Nijhoff Publishing.
Färe, R., Grosskopf, S., Lindgren, B. & Roos, P. (1992). Productivity changes in swedish pharmacies 1980-89: a nonparametric Malmquist approach. Journal of Productivity Analysis, *3*(1-2), 85-101.
Färe, R., Grosskopf, S., Norris, M. & Zhang, Z. (1994). Productivity growth, technical progress, and efficiency change in industrialized countries. America Economic Review, *84*(1), 66-83.
Farrell, M.J. (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society, Series A, 120, part III, 253-290.
Fried, H.; Lovell, C. A. K. (1999). Searching for the Zeds. In: The Georgia Productivity Workshop II, Univ. of Georgia.
Gomes, E.G. & Lins, M.P.E. (2008). Modelling undesirable *outputs* with zero sum gains data envelopment analysis models. Journal of the Operational Research Society, Vol. 59, n. 5, p. 616-623.
Hanushek, E. (1986). The economics of schooling: Production and Efficiency in Public Schools. Journal of Economic Literature, 24(3), 1141-1177.
Hanushek, E. (1989). The impact of Differential Expenditures on School Performance. Educational Researcher, 18(4), 45-63.
Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira- INEP. Censo Escolar e Sistema de Avaliação da Educação Básica (Saeb). (2005)**.** Retrieved July 10, 2009, from: http://www.inep.gov.br/basica/censo/Escolar/Sinopse
Marinho, E. & Barreto, F. (2000, dezembro). Avaliação do crescimento da produtividade e do progresso tecnológico dos estados do Nordeste com a fronteira de produção estocástica. Pesquisa e Planejamento Econômico, Rio de Janeiro, *30*(3), 427-452.
Marinho, E., Ataliba, F. & Lima F. (2001). Produtividade, variação tecnológica e variação de eficiência técnica das regiões e estados brasileiros. Anais do Encontro Nacional de Economia, Salvador, Brasil, 29,.
Muñiz, M. (2001). Introducción de variables de control en modelos DEA. In: A. Álvarez (Ed.). La Medición de la eficiencia y la productividad. Madrid: Ed. Pirámide,.
Orea, L. (2001). Medición y descomposición de la productividad. In: A. ÁLVAREZ (Ed.). La Medición de la eficiencia y la productividad. Madrid: Ed. Pirámide,.
Pedraja-Chaparro, F., Salina-Jiménez, J. & Suárez-Pandiello, J. (2001). La Medición de la eficiencia en el sector público. In: A. Álvarez (Ed.). La Medición de la eficiencia y la productividad. Madrid: Ed. Pirámide.
Ruggiero, J. (1996).Efficiency of Educational Production: An Analysis of New York School Districts. The Review of Economics and Statistics, Vol. 78, No. 3 (Aug.) , pp. 499-509.
Shephard, R.W. (1953). Cost and production functions. Princeton, NJ: Princeton University Press.